$$u=\frac{d_1d_4-d_2d_3\pm\sqrt{[(d_1d_4-d_2d_3)^2+4(d_1d_3-d_2^2)(d_2^2-d_2d_4)]}}{2(d_1d_3-d_2^2)}.$$

From (13) we find v. Then from (10) we find z. From (6), y, and from (1), x.

351. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Mich.

Solve, $y^2+yz+z^2=a^2\ldots(1)$.
$z^2+zx+x^2=b^2\ldots(2)$.
$x^2+xy+y^2=c^2\ldots(3)$.

I. Solution by J. A. COLSON, Searsport, Maine.

$b^2-c^2=(z-y)(x+y+z)$. $\therefore (b^2-c^2)x=(zx-xy)(x+y+z)$.

$c^2-a^2=(x-z)(x+y+z)$. $\therefore (c^2-a^2)y=(xy-yz)(x+y+z)$.

$a^2-b^2=(y-x)(x+y+z)$. $\therefore (a^2-b^2)z=(yz-zx)(x+y+z)$.

$\therefore (b^2-c^2)x+(c^2-a^2)y+(a^2-b^2)z=0$.

For convenience, put $b^2-c^2=f$, $c^2-a^2=g$, and $a^2-b^2=h$. Then $f+g+h=0$, and $fx+gy+hz=0$.

$$\therefore z=-\frac{fx+gy}{h}, \text{ and } x+y+z=x+y-\frac{fx+gy}{h}=\frac{(h-f)x+(h-g)y}{h}.$$

$$\therefore a^2-b^2=h=(y-x)(x+y+z)=(y-x)\frac{(h-f)x+(h-g)y}{h}.$$

$$\therefore h^2=(f-h)x^2+(g-f)xy+(h-g)y^2.$$

But from (3) we have $y^2=c^2-x^2-xy$.

Hence, $h^2=c^2(h-g)+(f+g-2h)x^2+(2g-f-h)xy=c^2(h-g)-3hx^2+3gxy$.

$$\therefore y=\frac{3hx^2+h^2+c^2(g-h)}{3gx}.$$

Substitute in (3), and we have

$$x^2+\frac{3hx^2+h^2+c^2(g-h)}{3g}+\frac{[3hx^2+h^2+c^2(g-h)]^2}{9g^2x^2}-c^2=0.$$

Hence, clearing of fractions and uniting, we have,

$$9(g^2+gh+h^2)x^4-3[c^2(2g^2-gh+2h^2)-h^2(g+2h)]x^2$$
$$+[h^2+c^2(g-h)]^2=0.$$

$$\therefore 36(g^2+gh+h^2)^2x^4-12(g^2+gh+h^2)[c^2(2g^2-gh+2h^2)-h^2(g+2h)]x^2$$
$$+[c^2(2g^2-gh+2h^2)-h^2(g+2h)]^2=[c^2(2g^2-gh+2h^2)-h^2(g+2h)]^2-4(g^2$$
$$+gh+h^2)[h^2-c^2(g-h)]^2=9c^4g^2h^2-6c^2g^2h^2(2g+h)-3g^2h^4.$$

$$\therefore 6(g^2+gh+h^2)x^2-[c^2(2g^2-gh+2h^2)-h^2(g+2h)]$$
$$=\pm gh\sqrt{[9c^4-6c^2(2g+h)-3h^2]}.$$

Giving g and h their original values, we have

$$6(a^4+b^4+c^4-b^2c^2-c^2a^2-a^2b^2)x^2=2b^6+2c^6-a^6+4a^4(b^2+c^2)$$
$$-5a^2(b^4+c^4)-5a^2(b^4+c^4)+b^2c^2(b^2+c^2-3a^2)$$
$$\pm(a^2-b^2)(c^2-a^2)\sqrt{[3(2b^2c^2+2c^2a^2+2a^2b^2-a^4-b^4-c^4)]}.$$

If k=the area of a triangle whose sides are a, b, and c, then $2b^2c^2+2c^2a^2+2a^2b^2-a^4-b^4-c^4=16k^2$.

Hence, $6(a^4+b^4+c^4-b^2c^2-c^2a^2-a^2b^2)x^2$

$$=2b^6+2c^6-a^6+4a^4(b^2+c^2)-5a^2(b^4+c^4)$$
$$b^2c^2(b^2+c^2-3a^2)\pm4(a^2-b^2)(c^2-a^2)k\sqrt{3}.$$

Hence, by permuting the letters a, b, c we can find the values of y^2 and z^2 from the two following equations:

$$6(a^4+b^4+c^4-b^2c^2-c^2a^2-a^2b^2)y^2=2c^6+2c^6+2a^6-b^6+4b^4(c^2+a^2)$$
$$-5b^2(c^4+a^4)+c^2a^2(c^2+a^2-3b^2)\pm4(b^2-c^2)(a^2-b^2)(k\sqrt{3}$$

and $6(a^4+b^4+c^4-b^2c^2-c^2a^2-a^2b^2)z^2=2a^6+2b^6-c^6+4c^4(a^2+b^2)$

$$-5c^2(a^4+b^4)+a^2b^2(a^2+b^2-3c^2)\pm4(c^2-a^2)(b^2-c^2)k\sqrt{3}.$$

II. Solution by ARTEMAS MARTIN, LL. D., Editor and Publisher, Mathematical Magazine, Washington, D. C.

From the square of the sum of the given equations, subtract twice the sum of their squares and extract the square root of one-third of the remainder; then

$$xy+yz+xz=\pm\tfrac{1}{3}\sqrt{[3(a^2+b^2+c^2)^2-6(a^4+b^4+c^4)]}\ldots(4).$$

Subtracting twice (1) from the sum of (4) added to the sum of the given equations,

$$2x(x+y+z)=b^2+c^2-a^2\pm\tfrac{1}{3}\sqrt{[3(a^2+b^2+c^2)^2-6(a^4+b^4+c^4)]}\ldots(5).$$

Subtracting twice (2) and twice (3) in succession from the same sum,

$$2z(x+y+z)=a^2+c^2-b^2\pm\tfrac{1}{3}\sqrt{[3(a^2+b^2+c^2)^2-6(a^4+b^4+c^4)]}\ldots(6),$$
$$2z(x+y+c)=a^2+b^2-c^2\pm\tfrac{1}{3}\sqrt{[3(a^2+b^2+c^2)^2-6(a^4+b^2+c^4)]}\ldots(7).$$

Add the three equations (5), (6), and (7); then

$$2(x+y+z)(x+y+z)=2(x+y+z)^2=a^2+b^2+c^2$$
$$\pm\sqrt{[3(a^2+b^2+c^2)^2-6(a^4+b^4+c^4)]}\ldots(8).$$

Extracting the square root of twice (8),

$$2(x+y+z)=\pm\sqrt{\{2(a^2+b^2+c^2)\pm2\sqrt{[3(a^2+b^2+c^2)^2-6(a^4+b^4+c^4)]}\}}\,..(9).$$

Dividing (5), (6), and (7) in succession by (9),

$$x=\frac{b^2+c^2-a^2\pm\frac{1}{3}\sqrt{[3(a^2+b^2+c^2)^2-(a^4+b^4+c^4)]}}{\pm\sqrt{\{2(a^2+b^2+c^2)\pm2\sqrt{[3(a^2+b^2+c^2)^2-(a^4+b^4+c^4)]}\}}},$$

$$y=\frac{a^2+c^2-b^2\pm\frac{1}{3}\sqrt{[3(a^2+b^2+c^2)^2-(a^4+b^4+c^4)]}}{\pm\sqrt{\{2(a^2+b^2+c^2)\pm2\sqrt{[3(a^2+b^2+c^2)^2-(a^4+b^4+c^4)]}\}}},$$

$$z=\frac{a^2+b^2-c^2\pm\frac{1}{3}\sqrt{[3(a^2+b^2+c^2)^2-(a^4+b^4+c^4)]}}{\pm\sqrt{\{2(a^2+b^2+c^2)\pm2\sqrt{[3(a^2+b^2+c^2)^2-(a^4+b^4+c^4)]}\}}}.$$

Also solved by A. H. Holmes, J. Scheffer, and V. M. Spunar.

For a number of different solutions of this problem, when the known quantities are not squared, see *The Mathematical Magazine*, published by Dr. Artemas Martin, Vol. II, pp. 141-144, and pp. 193-196. Ed. F.

GEOMETRY.

375. Proposed by C. N. SCHMALL, New York City.

From a point P on a circle there are drawn three chords PA, PB, PC. Show that the circles described on these chords as diameters intersect again in three collinear points.

I. Solution by S. G. BARTON, Ph. D., Clarkson School of Technology.

Take the point P as the origin of polar coördinates, and the diameter through P as the initial line. The coördinates of the points A, B, and C are, respectively, $2a\cos\alpha$, α; $2a\cos\beta$, β; $2a\cos\gamma$, γ; α, β, and γ being the vectorial angles.

The equations of the circles described upon the chords as diameters will be

$$\rho=2a\cos\alpha\cos(\theta-\alpha),$$
$$\rho=2a\cos\beta\cos(\theta-\beta),$$
$$\rho=2a\cos\gamma\cos(\theta-\gamma),$$

whence the coördinates of the points of intersection are

$$2a\cos\beta\cos\gamma,\ \beta+\gamma;\quad 2a\cos\gamma\cos\alpha,\ \gamma+\alpha;\quad 2a\cos\alpha\cos\beta,\ \alpha+\beta.$$

These points are all on the straight line whose equation is